Substitution

$$\int_a^b f(\varphi(x))\,\varphi'(x)\,dx \underset{\underset{\varphi(x)=u}{\uparrow}}{=} \int_{\varphi(a)}^{\varphi(b)} f(u)\,du$$

Bsp: (mit Anleitung)

(1) $$\int_2^5 (2x^2-3)^2 \cdot \cos(2x^2-3)\cdot x\,dx, \quad u = 2x^2-3, \quad \frac{du}{dx} = 4x \Rightarrow \text{„} du = 4x\,dx\text{"}$$

$$u(2) = 5 \qquad \text{„} dx = \frac{1}{4x}\,du\text{"}$$

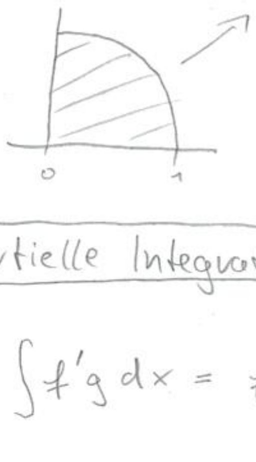
$$u(5) = 47$$

$$= \int_5^{47} u^2 \cdot \cos(u) \cdot x \cdot \frac{1}{4x}\,du = \int_5^{47} u^2 \cos(u) \cdot \frac{1}{4}\,du \underset{\text{P.I.}}{=} \ldots$$

(2) $$\int_2^3 \frac{1}{x\,\ln(x)^2}\,dx \underset{u=\ln(x)}{=} \int_{\ln(2)}^{\ln(3)} \frac{1}{u^2}\,du = -\left[\frac{1}{u}\right]_{\ln(2)}^{\ln(3)}$$

$$\frac{du}{dx} = \frac{1}{x} \Rightarrow du = \frac{1}{x}\,dx$$

(3) „Andersrum"



$$\int_0^1 \sqrt{1-x^2}\,dx \underset{x=\sin(t)}{=} \int_0^{\pi/2} \underbrace{\sqrt{1-\sin^2(t)}}_{=|\cos(t)|} \cdot \cos(t)\,dt = \int_0^{\pi/2} \cos^2(t)\,dt$$

$$\frac{dx}{dt} = \cos(t) \Rightarrow dx = \cos(t)\,dt$$

$$= \left[\frac{1}{2}t + \cos(t)\sin(t)\right]_0^{\pi/2} = \pi/4$$

$\sin(t_0) = 0 \Rightarrow$ z.B $t_0 = 0$

$\sin(t_1) = 1 \Rightarrow$ z.B $t_1 = \pi/2$

partielle Integration

$$\int f'g\,dx = fg - \int f g'\,dx$$

Bsp: (1) $$\int \underset{\downarrow}{x} \cdot \underset{\uparrow}{\cos(x)}\,dx = x \cdot \sin(x) - \int \sin(x)$$

$$= x \cdot \sin(x) + \cos(x)$$

(2) $$\int \underset{\downarrow}{x} \cdot \underset{\uparrow}{e^{\lambda x}}\,dx$$

$$= x \cdot \frac{1}{\lambda} e^{\lambda x} - \int \frac{1}{\lambda} \cdot e^{\lambda x}\,dx$$

$$= \frac{x \cdot e^{\lambda x}}{\lambda} - \frac{e^{\lambda x}}{\lambda^2}$$