Wdh:

$$\sum_{k=3}^{4} \frac{k+1}{2k} = \frac{3+1}{2\cdot 3} + \frac{4+1}{2\cdot 4} = \underline{\underline{\frac{4}{6} + \frac{5}{8}}}$$

Endindex

Startindex

Laufindex

Bsp:

(2) $\sum_{k=0}^{n} q^k = 1 + q + q^2 + q^3 + \dots = \frac{1-q^{n+1}}{1-q}$

$q \in \mathbb{R}$

(geometrische Summe)

z.B. $\sum_{k=0}^{63} 2^k = 1+2+4+8 \dots = \frac{1-2^{64}}{1-2} = 2^{64}-1$

$\approx 18{,}4\cdot 10^{9}$ Reiskörner

$\hat{=}\ 730 \cdot 10^{9}$ tonnen Reis

Wdh Potenzen

(1) $$\frac{(a^2 \cdot b^{-3})^2 \cdot (\sqrt[3]{3})^2 \cdot (1+a)}{(-2)^3 \cdot b^{-2} a^2 \cdot (1-a^2)}$$

$$= \frac{(a^2)^2 \cdot (b^{-3})^2 \cdot (3^{1/3})^2 \cdot \cancel{(1+a)}}{(-1)^3 (2)^3 \cdot b^{-2} a^2 \cdot (1-a)\cancel{(1+a)}}$$

$$= \frac{a^{2\cdot 2} \cdot b^{(-3)\cdot 2} \cdot 3^{1/3 \cdot 2}}{-1 \cdot 8 \cdot b^{-2} \cdot a^2 \cdot (1-a)}$$

$$= \frac{a^{4} \cdot b^{-6} \cdot 3^{2/3} \cdot b^2}{-8 \cdot a^{2} \cdot (1-a)} = \frac{b^{-6+2} \cdot \sqrt[3]{3^2}}{-8\cdot(1-a)} = \frac{b^{-4} \cdot \sqrt[3]{9}}{-8(1-a)}$$

$$= \underline{\underline{- \frac{\sqrt[3]{9}}{8(1-a)\cdot b^4}}}$$

② Sonnenintensität $1361 \frac{W}{m^2} \approx 1{,}3 \cdot 10^3 \frac{W}{m^2}$

$$1m^2 = 100 \cdot 100 \, cm^2 = 10^4 cm^2$$

$$1W = 10^3 mW$$

$$1{,}3 \cdot 10^3 \frac{W}{m^2} = 1{,}3 \cdot 10^3 \frac{10^3 mW}{10^4 cm^2}$$

$$= 1{,}3 \cdot 10^3 \cdot 10^3 \cdot 10^{-4} \frac{mW}{cm^2}$$

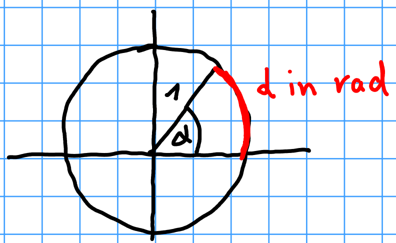
$$= 1{,}3 \cdot 10^2 \frac{mW}{cm^2} = 130 \frac{mW}{cm^2}$$

$$\hat{=} \text{ 3 LED auf } 1 cm^2$$

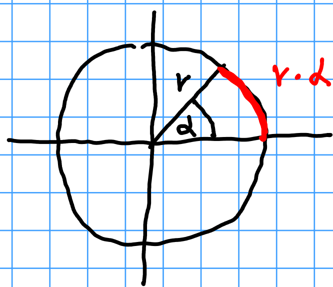
K1.7 Sinus und Kosinus (Trigonometrie)

Bogenmaß (rad) $\hat{=}$ Länge des Kreisbogens im Einheitskreis

Einheitskreis



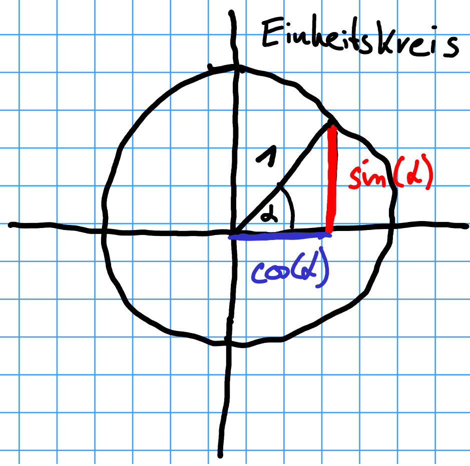
Kreis mit Radius r



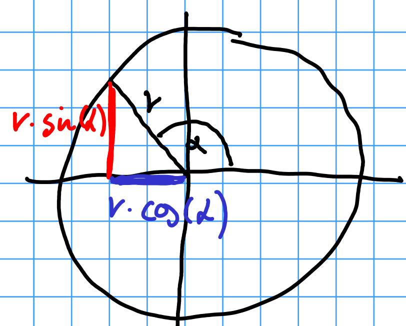
Winkel in Grad	360°	180°	90°	45°	a°
Winkel in rad	2π	π	$\pi/2$	$\pi/4$	$\frac{360}{a} \cdot 2\pi$

Sinus / Kosinus

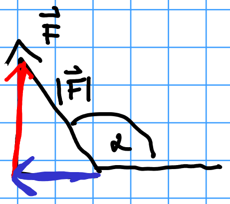
Einheitskreis



Kreis mit Radius r

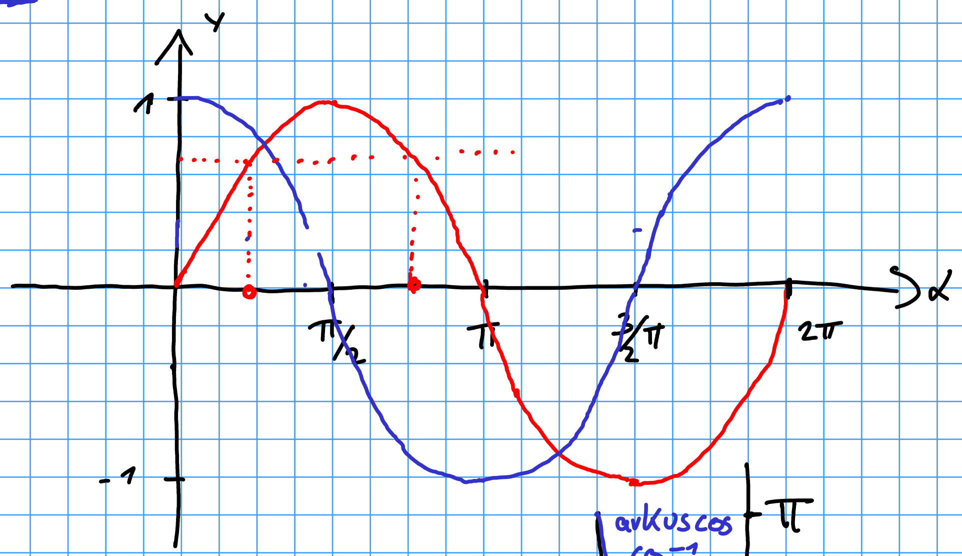


Bsp:

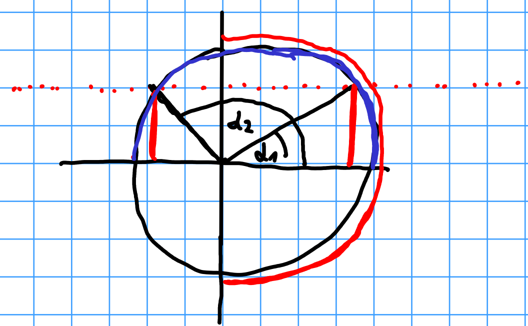


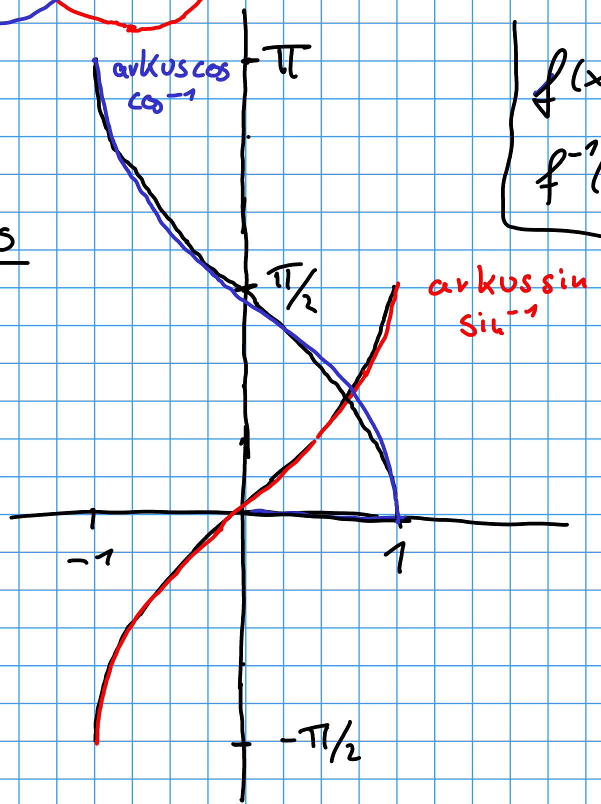
$F_y = \sin(\alpha) \cdot |\vec{F}|$

$F_x = \cos(\alpha) \cdot |\vec{F}|$



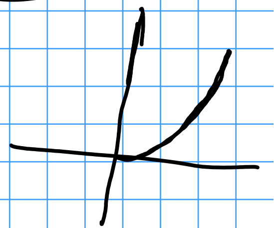
Umkehrfkt. Arkussinus, Arkuscos





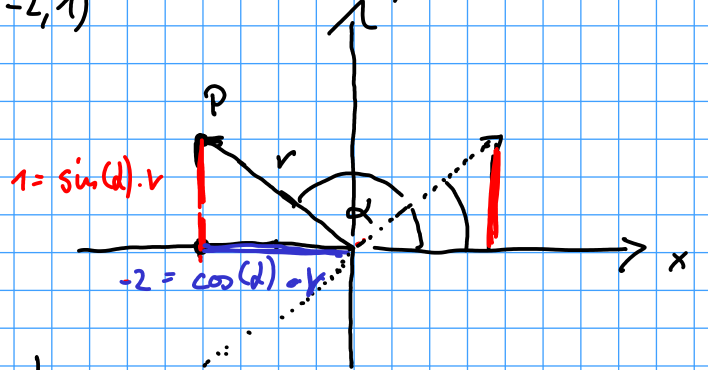
$f(x) = x^2$

$f^{-1}(y) = \sqrt{y}$



Bsp: Polarwinkel eines Punktes

$P = (-2, 1)$



gesucht α

$r = \sqrt{(-2)^2 + 1^2} = \sqrt{5}$

$$\left.\begin{aligned} 1 &= \sin(\alpha)\cdot\sqrt{5} \\ -2 &= \cos(\alpha)\cdot\sqrt{5} \end{aligned}\right\rangle \quad \begin{aligned} \sin(\alpha) &= \frac{1}{\sqrt{5}} \qquad (1) \\ \cos(\alpha) &= -\frac{2}{\sqrt{5}} \qquad (2) \end{aligned}$$

$(1) \Rightarrow \quad \arcsin\left(\frac{1}{\sqrt{5}}\right)$

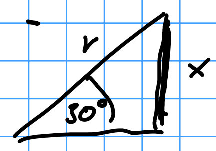
$\approx 0{,}46$

$\hat{\approx} 26{,}7^\circ$

NR: $\frac{0{,}46}{2\pi}\cdot 360^\circ = 26{,}7^\circ$

$\Rightarrow \quad \alpha = \pi - 0{,}46 \approx \underline{2{,}68}$

$\hat{=} \underline{\underline{153{,}3^\circ}}$



TR deg $\Rightarrow$ $x = \cos(30)\cdot r$

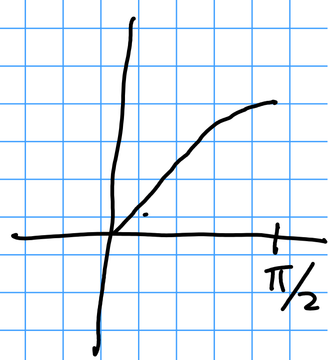
TR rad $\Rightarrow$ $x = \cos(\pi/6)\cdot r$

(2) $\quad \arccos\left(-\frac{2}{\sqrt{5}}\right) \approx \underline{\underline{2{,}68}} \hat{=} \underline{\underline{153{,}3^\circ}}$

$\approx 0{,}85\cdot\pi$

Wichtige exakte Werte von sin/cos

α in rad	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
α in °	0	30°	45°	60°	90°
$\cos(\alpha)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$\sqrt{1}/2$	0
$\sin(\alpha)$	0	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$1=\sqrt{4}/2$



K2 Analysis

K2.1. reelle Funktionen

Was ist eine Funktionen?

$$\text{Input} \xrightarrow{\text{Funktion}} \text{Output}$$

Eine reelle Fkt. $f: \underbrace{D_f}_{\text{Def. bereich}} \longrightarrow \underbrace{\mathbb{R}}_{\text{Bildbereich}}$, $D_f \subseteq \mathbb{R}$

ist eine Zuordnung, bei der jede Zahl $x \in D_f$ genau eine Zahl $y = f(x)$ zugeordnet ist

Bsp: ① $f: \{1, 2, 3\} \longrightarrow \mathbb{R}$

mit $1 \longmapsto 5$, $2 \longmapsto -3$, $3 \longmapsto 100$

Also z.B. $f(1) = 5$, $f(2) = -3$ $\quad W_f = \{5, -3, 100\}$

② $g: \mathbb{R} \longrightarrow \mathbb{R}$

$x \longmapsto 2 \cdot x - 1$ $\quad W_g = \mathbb{R}$

z.B. $g(1) = 1$, $g(6) = 11$

③ $h: \mathbb{R} \rightarrow \mathbb{R}$ $\quad W_h = \mathbb{R}_{\geq 0}$

$x \mapsto x^2$ $\quad$ z.B. $g(2) = 4$ $\quad = [0, \infty)$

$g(-2) = 4$

Wertebereich:

Ist $f: D_f \rightarrow \mathbb{R}$ eine Funkt, dann heißt $W_f := \{ f(x) \in \mathbb{R} \mid x \in D_f \}$

Wertebereich von f

Einschub:

Intervalle

$$[a,b] = \{ x \in \mathbb{R} \mid a \leq x \leq b \}$$

$$(a,b) = \{ x \in \mathbb{R} \mid a < x < b \}$$

$$\shortparallel$$

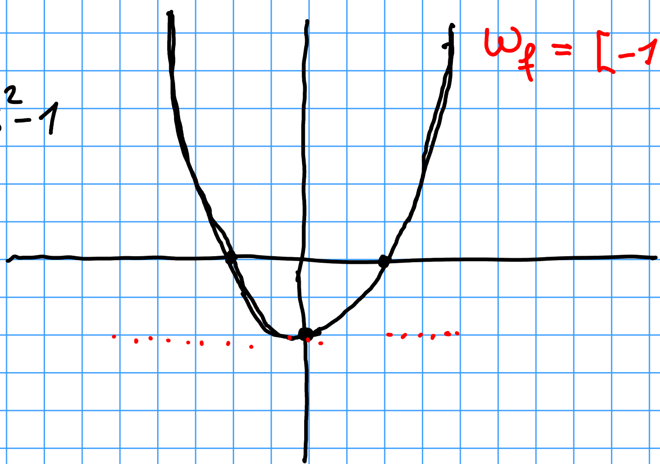
$$]a,b[= \{ x \in \mathbb{R} \mid a < x < b \}$$

Graphen

In einem xy-KS tragen wir die Punkte $(x, f(x))$ ein, $x \in D_f$

Bsp:

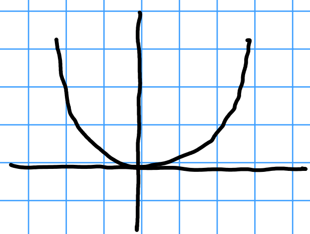
$$f: \mathbb{R} \to \mathbb{R}$$
$$x \mapsto x^2 - 1$$



$W_f = [-1, \infty)$

Graphtransformationen

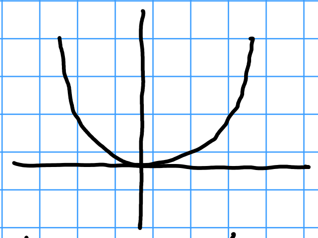
$$f(x) = x^2$$



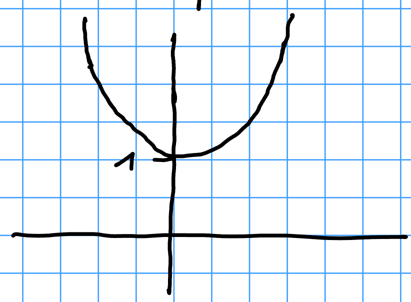
f

Graphtransformationen

$f(x) = x^2$

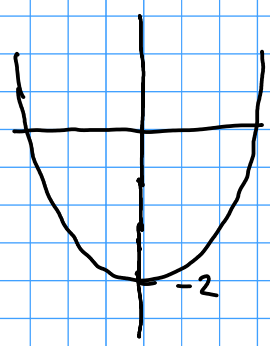


$f(x) = x^2 + 1$

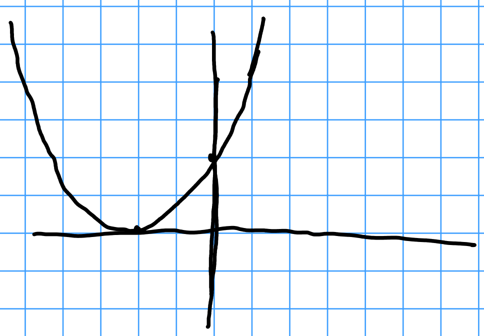


Verschiebung nach oben

$f(x) = x^2 - 2$

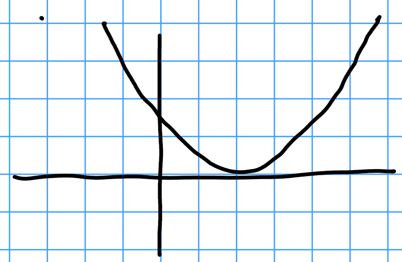


Verschiebung nach unten



$g(x) = f(x+1) = (x+1)^2$

Verschiebung nach links



$g(x) = f(x-1) = (x-1)^2$

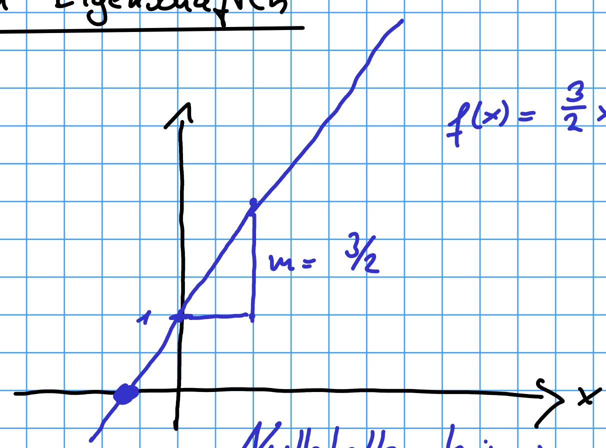
Verschiebung nach rechts

K2.2 Bekannte Funktionen und Eigenschaften

① Lineare Fkt

$f: \mathbb{R} \to \mathbb{R}$

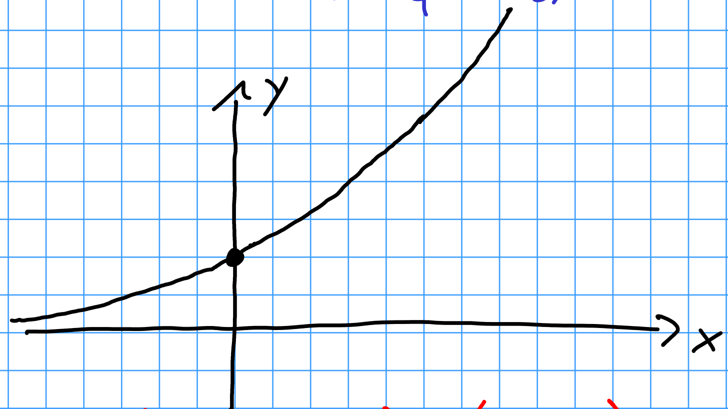
$x \mapsto mx + c$



$f(x) = \frac{3}{2}x + 1$

Nullstelle bei $x = -2/3$

weil $f(-2/3) = 0$



② Exponentialfkt.

$$\exp: \mathbb{R} \to \mathbb{R}$$
$$x \mapsto e^x$$

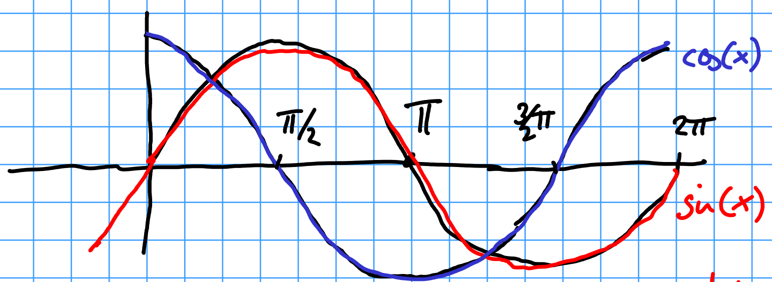
$e \approx$ eulersche Zahl $\approx 2{,}71$

monoton steigend $W_f = (0, \infty)$

stetig

③ Sinusfkt / Kosinusfkt

$$\sin: \mathbb{R} \to \mathbb{R} \qquad\qquad \cos: \mathbb{R} \to \mathbb{R}$$
$$x \mapsto \sin(x) \qquad\qquad x \mapsto \cos(x)$$



$W_{\cos} = [-1, 1]$

$W_{\sin} = [-1, 1]$

nicht monoton steigend

stetig

④ Gebrochenrationale Fkt.

$$f: D_f \to \mathbb{R}$$
$$x \longmapsto \frac{N(x)}{Z(x)}$$

wobei $N(x)$, $Z(x)$ Polynome sind

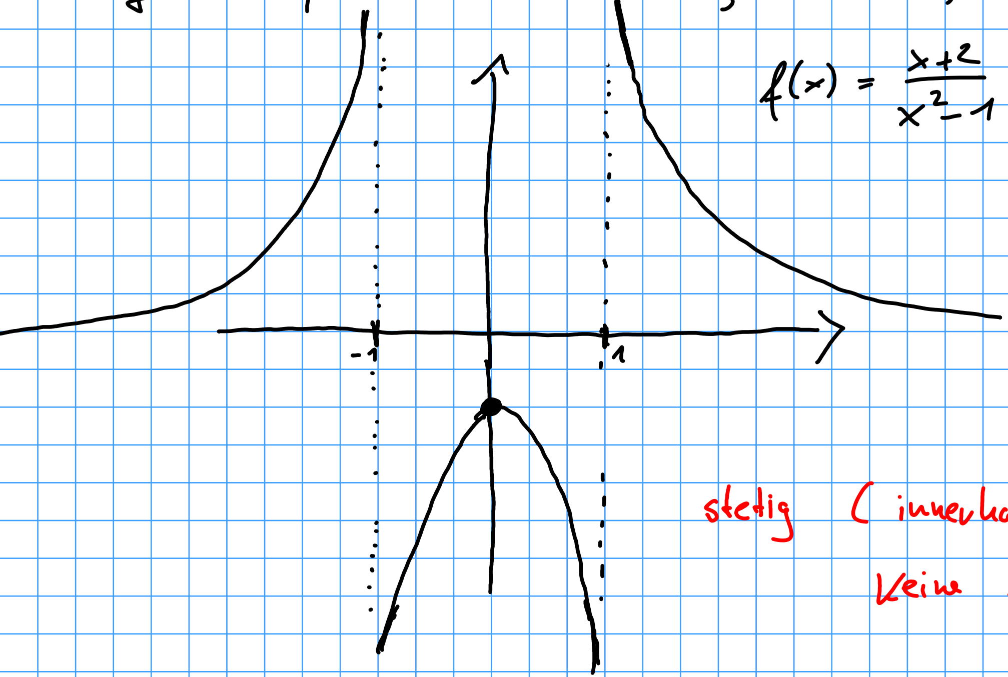
$$Z(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$$
$$N(x) = b_m x^m + \dots + b_0$$

zB. $Z(x) = x + 2$

$N(x) = x^2 - 1 = (x-1)(x+1)$

$$f(x) = \frac{x+2}{x^2-1}$$

$D_f = \mathbb{R} \setminus \{x \in \mathbb{R} \mid x^2 - 1 = 0\} = \mathbb{R} \setminus \{-1, 1\}$



$$f(x) = \frac{x+2}{x^2-1} = \frac{x\,(1+\frac{2}{x})}{x^2\,(1-\frac{1}{x^2})}$$

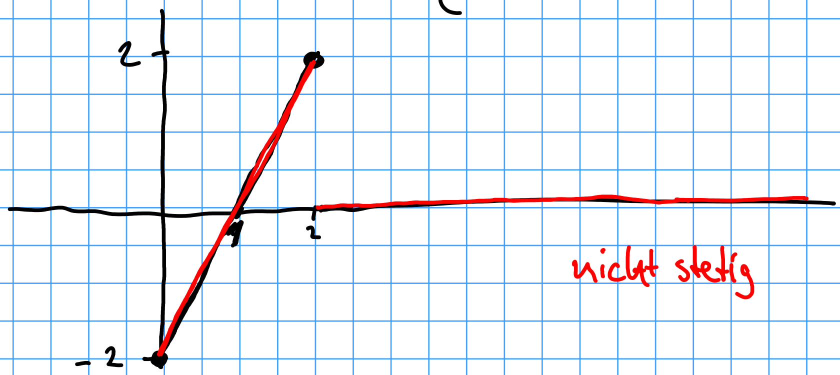
$$= \frac{1}{x} \cdot \left(\frac{1+\frac{2}{x}}{1-\frac{1}{x^2}}\right) \xrightarrow[x\to\infty]{} 0 \cdot 1 = 0$$

stetig (innerhalb des Def bereich keine Sprünge)

⑤ Stückweise def. Funktionen

$$f(x): [0,\infty) \to \mathbb{R}$$

$$x \mapsto \begin{cases} 2x-2 & 0 \le x \le 2 \\ 0 & 2 < x \end{cases}$$



nicht stetig

f monoton steigend $\Leftrightarrow$ $x_1 \le x_2 \Rightarrow f(x_1) \le f(x_2)$

f stetig $\Leftrightarrow$ „Schaubild hat keine Sprünge"