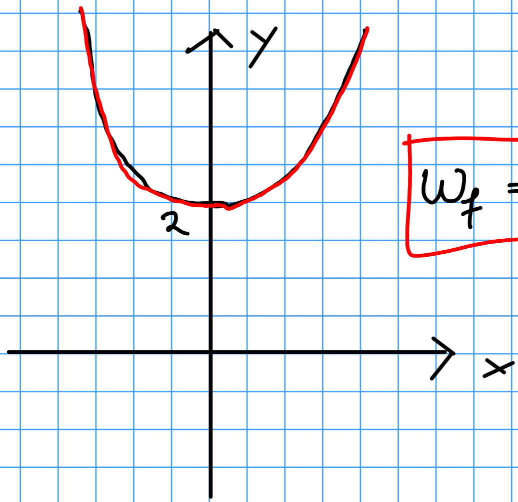
Aufgabe 1

(a) $f = \mathbb{R} \to \mathbb{R}$

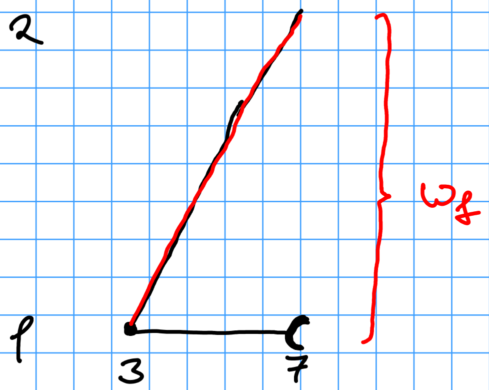
$x \mapsto x^2 + 2$



$\boxed{W_f = [2, \infty)}$

(b) $f : [3,7) \to \mathbb{R}$

$x \mapsto 3x - 2$



f steigt monoton

mit $f(3) = 7$

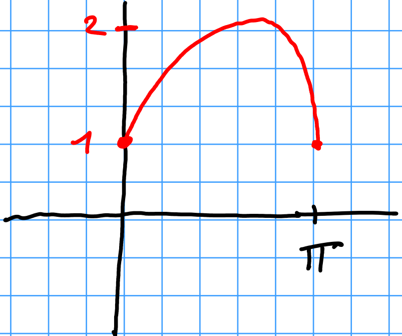
$f(7) = 19$

$\boxed{W_f = [7, 19)}$

19 wird nicht angenommen auf $[3,7)$

(c) $f : [0, \pi] \to \mathbb{R}$

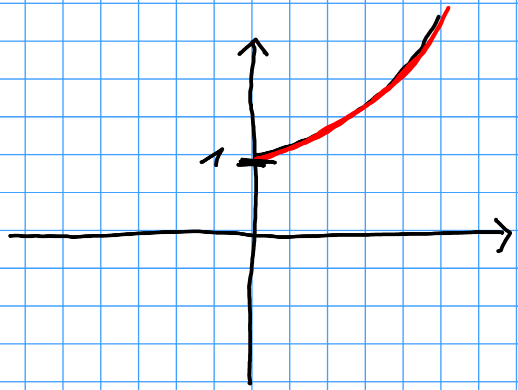
$x \mapsto \sin(x) + 1$



$\boxed{W_f = [0, 1]}$

(d) $f: [0,\infty] \to \mathbb{R}$

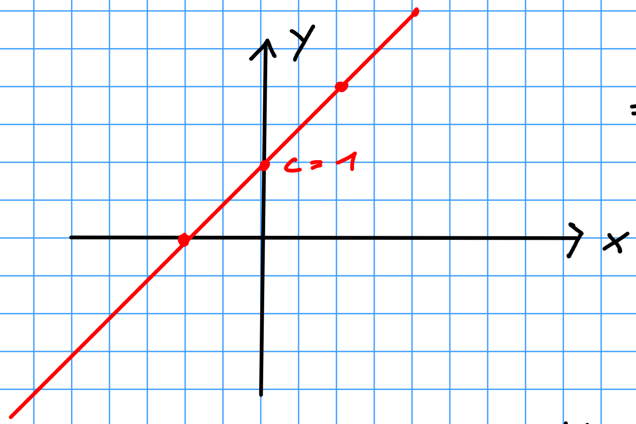
$x \mapsto e^x$



$\boxed{W_f = [1,\infty)}$

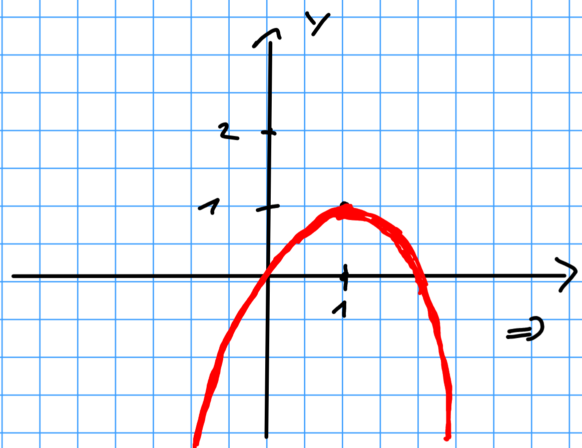
Aufgabe 2

(a) $f(x) = x + 1$



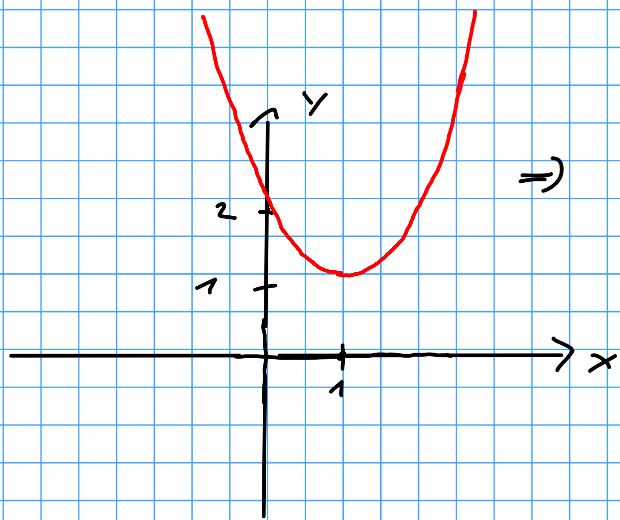
$\Rightarrow$ Graph 3

(b) $f(x) = -(x-1)^2 + 1$



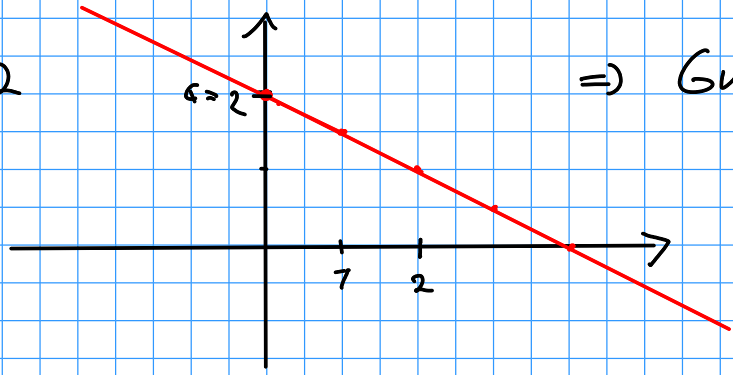
$\Rightarrow$ Graph 6

(c) $f(x) = (x-1)^2 + 1$



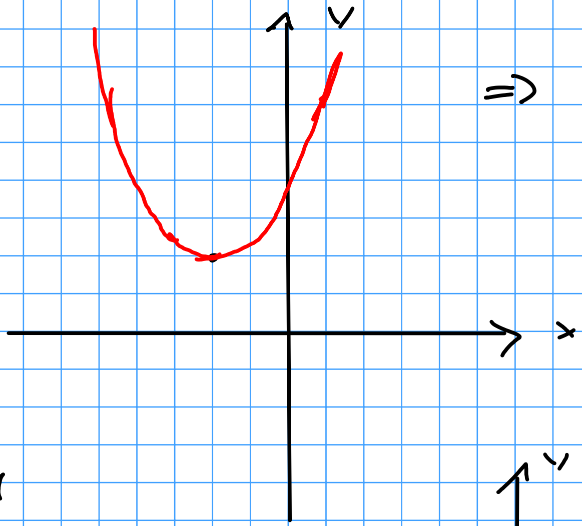
$\Rightarrow$ Graph 4

(d) $f(x) = -\frac{1}{2}x + 2$



$\Rightarrow$ Graph 1

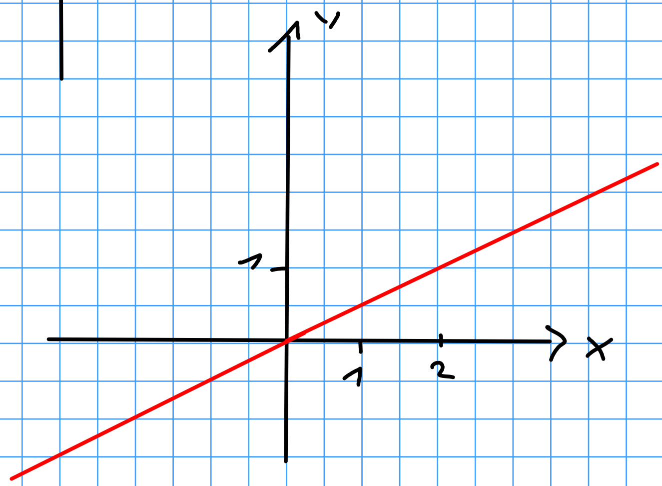
(e) $f(x) = (x+1)^2 + 1$



$\Rightarrow$ Graph 2

(f) $f(x) = \frac{1}{2}(x-2) + 1$

$= \frac{1}{2}x - 1 + 1 = \frac{1}{2}x$



$\Rightarrow$ Graph 5

Aufgabe 3

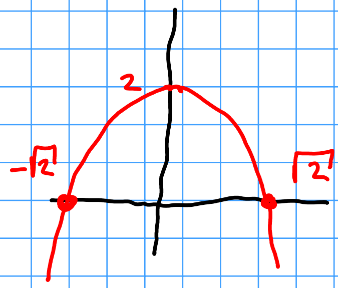
(i) Def. Menge endlich $\Rightarrow$ W. Menge endlich
ist wahr, denn es kann nicht mehr Elemente in der W. Menge geben als in der Def. Menge

(ii) Def. Menge unendlich $\Rightarrow$ W. Menge unendlich

falsch z.B. $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto 3$ hat unendliche Def. Menge $\mathbb{R}$ aber $W_f = \{3\}$

(iii) $f: \mathbb{R} \to \mathbb{R}$
$x \mapsto -x^2+2$ hat genau eine Nullstelle

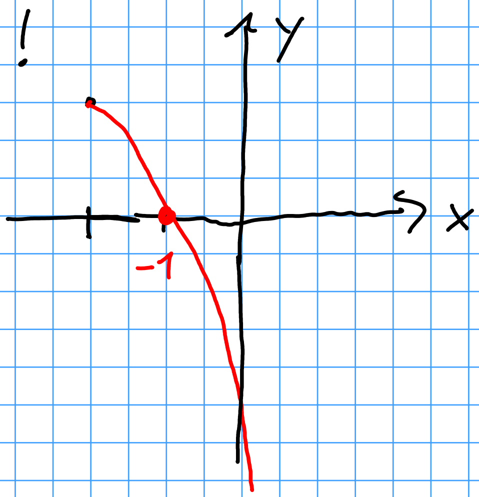
falsch



f hat zwei Nullstellen $-\sqrt{2}$, $\sqrt{2}$

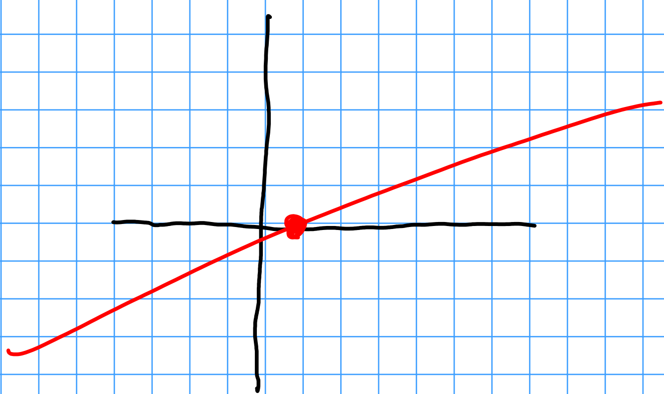
(iv) $f: [-2, \infty) \to \mathbb{R}$
$x \mapsto (x+2)^2 - 1$ hat genau eine Nst.

wahr!



f hat die Nst $x = -1$

(v) Eine lineare Fkt hat immer genau eine Nst wenn $m \neq 0$



wahr! $f(x) = mx + c = 0$

$\Leftrightarrow \quad mx = -c$

$\underset{m \neq 0}{\Leftrightarrow} \quad x = -c/m$

f hat die Nst $-c/m$

(VI) Der Graph der Funktion $f(x) = 2x+1$ ist der Graph von $g(x) = 2(x-1)+1$ um zwei nach oben verschoben.

wahr! $f(x) = 2x+1 = 2x+1-2+2$

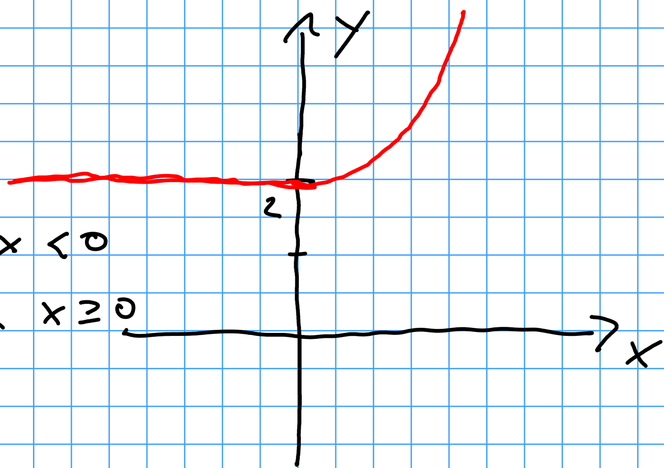
$$= 2(x-1)+1+2$$

$$= g(x)+2$$

(VII)

$$f: \mathbb{R} \to \mathbb{R}$$

$$x \mapsto \begin{cases} 0 & x < 0 \\ x^2+2 & x \geq 0 \end{cases}$$



wahr!

(VIII) Für f aus (VII) gilt $f(-3) = f(3)+2$

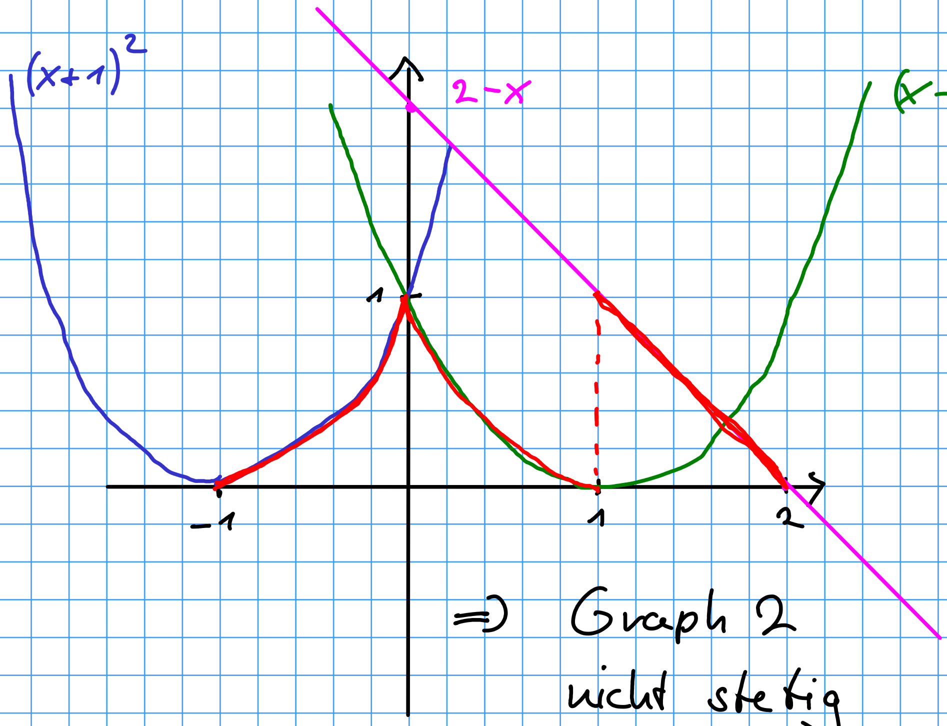
falsch! $f(-3) = 0$ also $f(-3) \neq f(3)+2$.

$f(3) = 3^2+2 = 11$

Aufgabe 4

(a) $f: [-1,2] \to \mathbb{R}$

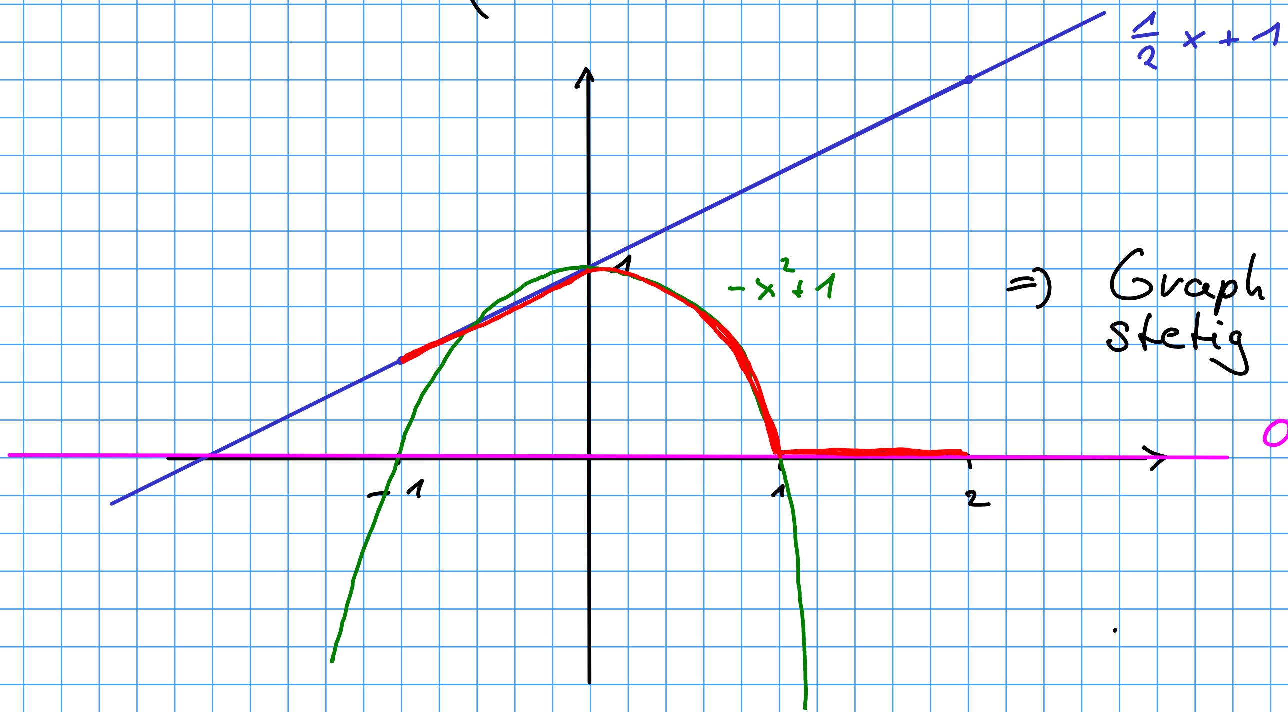
$$x \mapsto \begin{cases} (x+1)^2 & -1 \leq x \leq 0 \\ (x-1)^2 & 0 \leq x \leq 1 \\ 2-x & 1 < x \leq 2 \end{cases}$$



$\Rightarrow$ Graph 2 nicht stetig

(b) $f: [-1,2] \to \mathbb{R}$

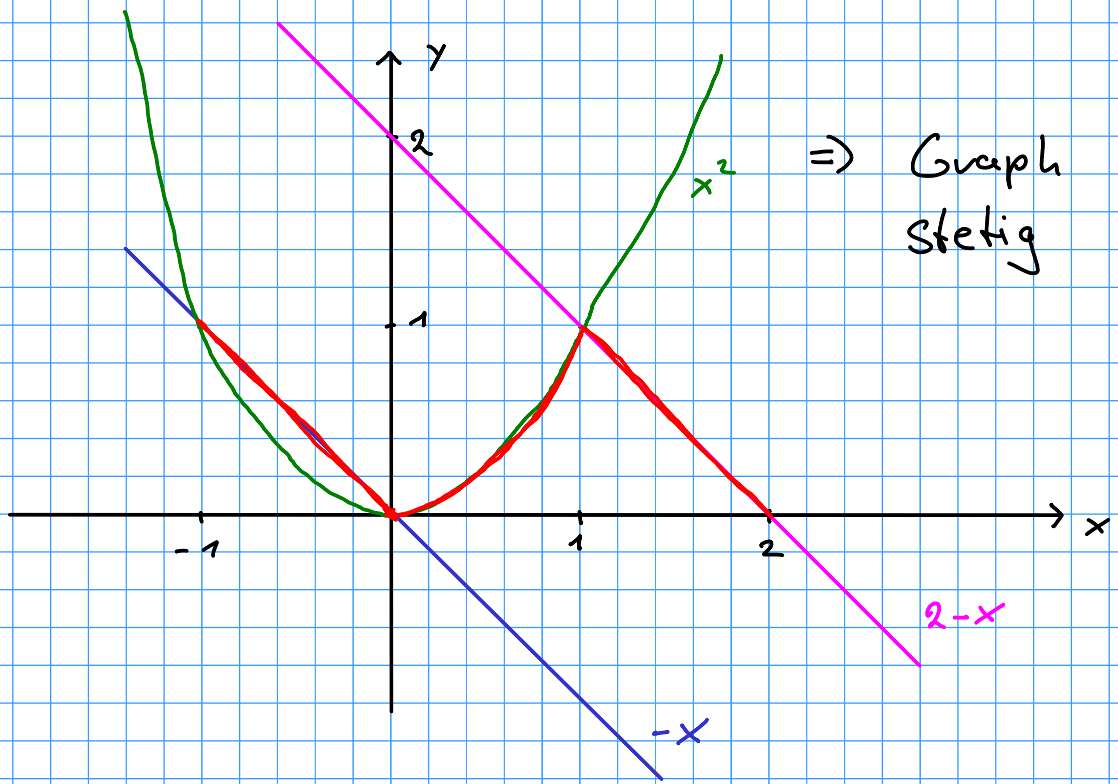
$$x \mapsto \begin{cases} \frac{1}{2}x+1 & -1 \leq x \leq 0 \\ -x^2+1 & 0 < x \leq 1 \\ 0 & 1 < x \leq 2 \end{cases}$$



$\Rightarrow$ Graph 1 stetig

(c) $f: [1,2] \to \mathbb{R}$

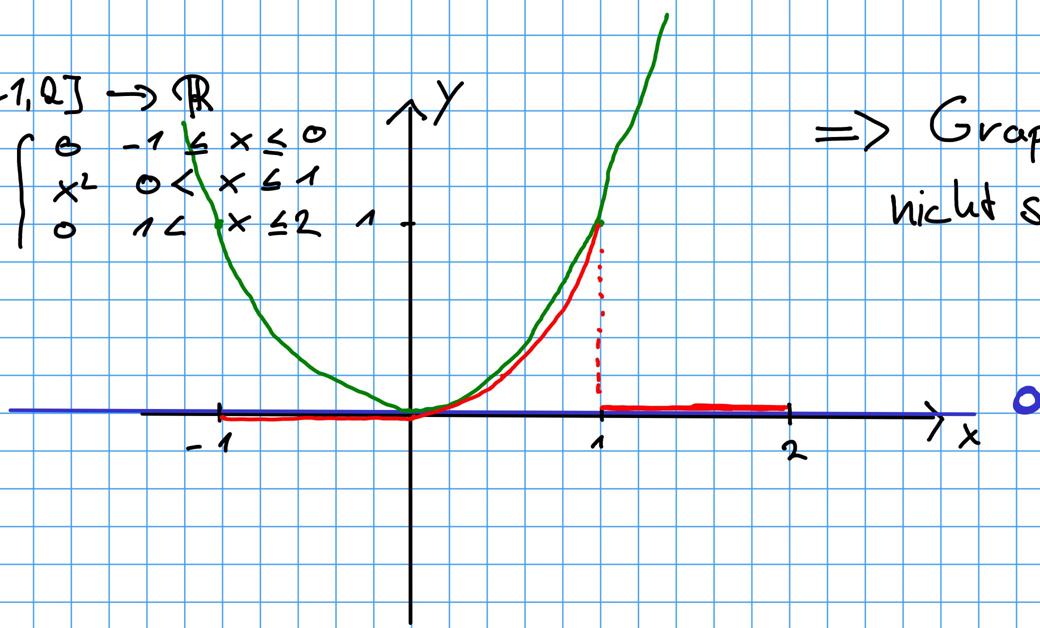
$$x \mapsto \begin{cases} -x & -1 \leq x \leq 0 \\ x^2 & 0 < x \leq 1 \\ 2-x & 1 < x \leq 2 \end{cases}$$



$\Rightarrow$ Graph 4 stetig

(d) $f: [-1,2] \to \mathbb{R}$

$$x \mapsto \begin{cases} 0 & -1 \leq x \leq 0 \\ x^2 & 0 < x \leq 1 \\ 0 & 1 < x \leq 2 \end{cases}$$



$\Rightarrow$ Graph 3 nicht stetig